May 2020 Activity Solutions

Warm-Up!

1. We are told that Mary chooses an integer from 1 to 100, inclusive that is a multiple of 4. There are 100/4 = 25 integer multiples of 4 from 1 to 100 that Mary could choose. Of those integer multiples of 4, there are 5 perfect squares: 4, 16, 36, 64, 100. Therefore, the probability that the multiple of 4 that Mary chooses is a perfect square is 5/25 = **1/5**.

2. Lets start by finding the probability that Arthur first draws the yellow ball numbered 3. Since there are a total of 20 balls in the bag, the probability is 1/20. Now we need to find the probability that Arthur next draws a red ball after the yellow ball numbered 3 has been drawn and not replaced. There are now 19 balls in the bag, 10 of which are red. So, the probability is 10/19. Therefore, the probability that Arthur draws a red ball from the bag after drawing the yellow ball numbered 3, without replacement, is 1/20 × 10/19 = **1/38**.

3. Let r represent the number of red balls in the bag. The probability of randomly drawing two red balls would be $(r/10) \times (r - 1)/9 = r(r - 1)/90$, which we are told equals 1/15. So we have the proportion $r(r - 1)/90 = 1/15$. Cross-multiplying, we get $15r(r - 1) = 90$, and $r(r - 1) = 6$. We are looking for consecutive numbers $r - 1$ and r whose product is 6. That means $r = 3$ and $r - 1 = 2$. So, before removing two red balls, the number of red balls in the bag was **3** balls.

4. (a) There are **0** units cubes with more than three blue faces.
   (b) Each of the **8** unit cubes at a corner of the original cube has exactly three blue faces.
   (c) There are 3 unit cubes along each of the 12 edges of the original cube, for a total of 3 × 12 = **36** unit cubes that each have exactly two blue faces.
   (d) On each of the 6 faces of the original cube, there are 9 cubes not along an edge, for a total of 6 × 9 = **54** unit cubes that each have exactly one blue face.
   (e) There are 3 × 3 × 3 = **27** unit cubes not on a face of the original cube, each with no blue faces.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. The probability that the group of three students is entirely girls is 12/20 × 11/19 × 10/18 = 11/57. Any other possibility must include at least one boy, so the probability of having at least one boy in the group is 1 – 11/57 = **46/57**.

6. There are four possibilities when two fair coins are tossed: HH, HT, TH and TT. If TT occurs, the coins are tossed again. In the other three cases, at least one coin lands heads up, but only one of these three involves two coins landing heads up. Therefore, the probability is **1/3**.

7. There are two combinations of five digits that have a sum of 43: 9, 9, 9, 9, 7 and 9, 9, 9, 8, 8. There are 5!/(4! × 1!) = 5 arrangements of the digits of 99997. There are 5!/(3! × 2!) = 10 arrangements of the digits of 99988. Thus, there are a total of 5 + 10 = 15 five-digit numbers whose digits have a sum of 43. In order for a number to be evenly divisible by 11, the difference between the sum of the odd-numbered digits and the sum of the even-numbered digits, counted from right to left, must be a multiple of 11. For example, 99979 is evenly divisible by 11 because (9 + 9 + 9) – (7 + 9) = 27 – 16 = 11, which is, of course, a multiple of 11. On the other hand, 99997 is not evenly divisible by 11 because (7 + 9 + 9) – (9 + 9) = 25 – 18 = 7, which is not a multiple of 11. Based on this rule, we see that 97999 and 98989 are the only other possible arrangements that form a number that is evenly divisible by 11. Therefore, the probability is 3/15 = **1/5**.

8. This problem might best be explained using tree diagrams. Figure 1 is a tree diagram that shows all of the possible outcomes when the two-headed coin is flipped three times. All of the 8 outcomes are the desired outcome HHH. Figure 2 is a tree diagram that shows all of the possible outcomes when the fair coin is flipped three times. Only 1 of the outcomes is the desired outcome HHH. Of the 8 + 1 = 9 ways to achieve the desired outcome, 8 of them are the result of flipping the two-headed coin. Therefore, the probability is **8/9**.

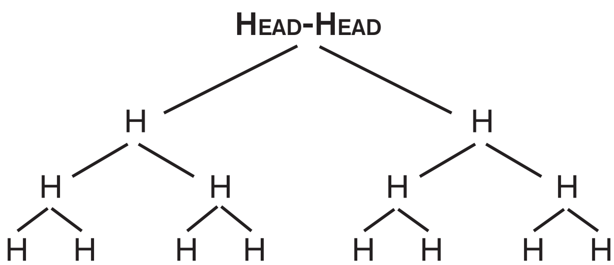

Figure 1

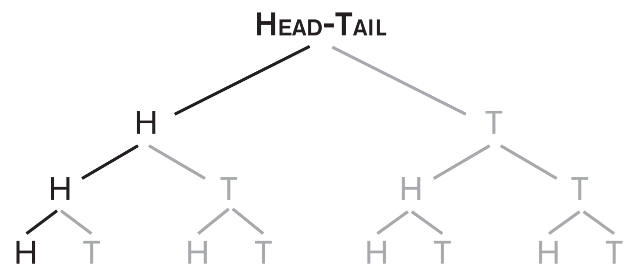

Figure 2